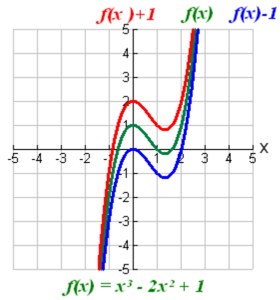
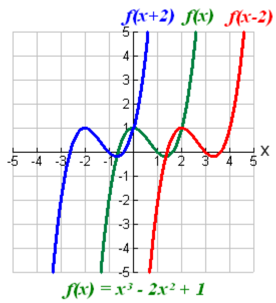
# Review: Composite Functions - 9/21/16

## 1 Transformations of Functions

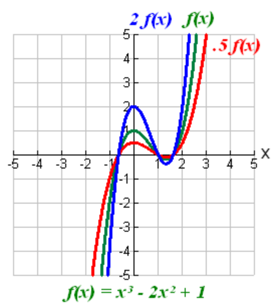
**Suppose $c > 0$. Then $y = f(x)+c$ shifts the graph of $f(x)$ $c$ units upwards, and $y = f(x)-c$ shifts the graph of $f(x)$ $c$ units downwards.**



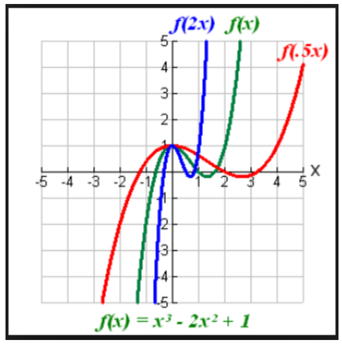

**Suppose $c > 0$. Then $y = f(x + c)$ shifts the graph of $f(x)$ $c$ units to the left, and $y = f(x - c)$ shifts the graph of $f(x)$ $c$ units to the right.**



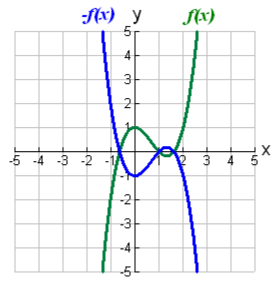

**Suppose $c > 1$. Then $y = cf(x)$ stretches the graph vertically by a factor of $c$, and $y = \frac{1}{c}f(x)$ shrinks the graph vertically by a factor of $c$.**



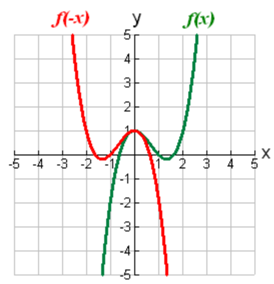

**Suppose $c > 1$. Then $y = f(cx)$ shrinks the graph horizontally by a factor of $c$, and $y = f(\frac{1}{c}x)$stretches the graph horizontally by a factor of $c$.**




$y = -f(x)$ **reflects the graph over the** $x$ **axis.**




$y = f(-x)$ **reflects the graph over the** $y$ **axis.**




| **Function** | **Action** | **Domain** | **Range** |
|---|---|---|---|
| $f(x)$ | none | $[a, b]$ | $[d, e]$ |
| $f(x) + c$ | translate $c$ units up | $[a, b]$ | $[d + c, e + c]$ |
| $f(x) - c$ | translate $c$ units down | $[a, b]$ | $[d - c, e - c]$ |
| $f(x + c)$ | translate $c$ units left | $[a - c, b - c]$ | $[d, e]$ |
| $f(x - c)$ | translate $c$ units right | $[a + c, b + c]$ | $[d, e]$ |
| $cf(x)$ | stretch vertically by a factor of $c$ | $[a, b]$ | $[cd, ce]$ |
| $\frac{1}{c}f(x)$ | shrink vertically by a factor of $c$ | $[a, b]$ | $[\frac{d}{c}, \frac{e}{c}]$ |
| $f(cx)$ | shrink horizontally by a factor of $c$ | $[\frac{a}{c}, \frac{b}{c}]$ | $[d, e]$ |
| $f(\frac{1}{c}x)$ | stretch horizontally by a factor of $c$ | $[ca, cb]$ | $[d, e]$ |
| $-f(x)$ | reflect over $x$ axis | $[a, b]$ | $[-e, -d]$ |
| $f(-x)$ | reflect over $y$ axis | $[-b, -a]$ | $[d, e]$ |

**Practice Problems**

1. Let $f$ by a function with domain $[3, 7]$ and range $[-1, 5]$. What is the domain and range of:

   (a) $-f(x + 4)$

(b) $3f(x) - 7$

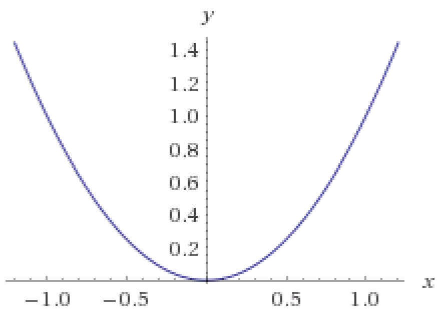
(c) $f(-x-3)$

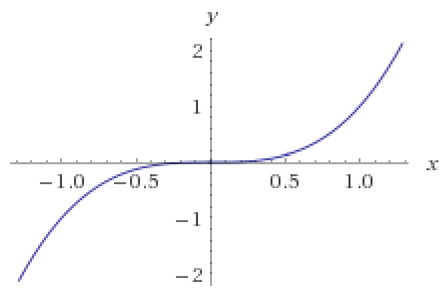
2. Sketch the graph of $f(x) = 2(x+3)^2 + 4$.

3. Sketch the graph of $g(x) = -x^3 - 2$.

## 2 Some pictures of $x^a$

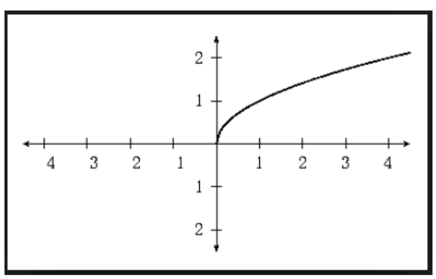
- $a$ is a positive integer

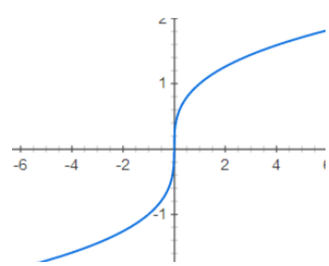



$x^n$ where $n$ is even $\qquad$ $x^n$ where $n$ is odd

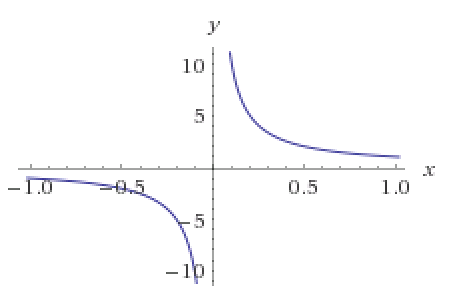
- $a = \frac{1}{n}$ where $n$ is a positive integer




$\sqrt[n]{x}$ where $n$ is even $\qquad$ $\sqrt[n]{x}$ where $n$ is odd

- $a = -1$




$\frac{1}{x}$

# 3 Composition

**Example 3.0.1** *Let* $h(x) = (4x+3)^3$. *Write it as a composition of two functions. Here* $h(x) = (f \circ g)(x)$ *where* $f(x) = x^3$ *and* $g(x) = 4x + 3$.

**Example 3.0.2** *Let* $h(x) = (x^2 - 4x + 4)^3$. *We can either break this into* $h(x) = (f \circ g)(x)$ *where* $f(x) = x^3$ *and* $g(x) = x^2 - 4x + 4$, *or we can write it as* $h(x) = (f \circ a \circ b)(x)$ *where* $f(x) = x^3$, $a(x) = x^2$, *and* $b(x) = x - 2$.

**Practice Problems**

1. Write $h(x) = (x^2 - 4x + 4)^2$ as a composition of two functions. Now try coming up with a different set of two functions that also works.
2. Write $h(x) = \sqrt[4]{(x^2 + 6x + 9)^3}$ as a composition of two functions. As a composition of three functions. As a composition of four functions.